A nonextensive thermodynamical equilibrium approach in $e^+e^- \rightarrow hadrons$

I. Bediaga, E. M. F. Curado and J.M. de Miranda

Centro Brasileiro de Pesquisas Físicas,

Rua Xavier Sigaud 150, 22290-180 – Rio de Janeiro, RJ, Brazil

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Abstract

We show that the inclusion of long distance effect, expected in strong interactions, through a nonextensive thermodynamical approach is able to explain the experimental distribution of the transverse momentum of the hadrons with respect to the jet axis (p_t) $e^+e^- \rightarrow$ *hadron* reaction. The observed deviation from the exponential behavior, predicted by the Boltzmann-Gibbs thermodynamical treatment, is automatically recovered by the nonextensive Tsallis statistics used here. We fitted the observed p_t spectrum in the range of 14 GeV to 161 GeV and obtained, besides a good fit, the theoretical important fact that the temperature becomes independent of the primary energy.

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1 Introduction

The global structure of multiple hadroproduction in e^+e^- annihilation, has been well understood with remarkable results concerning the total cross section and angular distribution calculations. Perturbative Quantum Electrodynamics and perturbative Quantum Chromodynamics provide a good description of the initial process involving short distances $e^+e^- \rightarrow$ quark-antiquark $(q\bar{q})$ and $e^+e^- \rightarrow$ quark-antiquark-gluon $(q\bar{q}g)$ interactions, main responsible for the global characteristics of this kind of process. The quarks produced off-shell initialize a cascade of several quarks and gluons with complex interactions themselves. This stage is in general described in probabilistic terms, based on leading-log Quantum Chromodynamics approximation. The outgoing colored partons are transformed into color singlet hadrons through the soft hadronization process, forming a jet of particles traveling approximately in the initial parton direction. However, hadronization require the Quantum Chromodynamics in the soft regime, where the coupling constant become large and there is no easy way to understand it from first principles.

For more than a decade several different models making use of Monte Carlo techniques were developed to describe such processes [1, 2, 3]. These methods avoid the microscopic complexity by being able to define probability distributions for each stage of the jet evolution. In these approaches, all known aspects involving Quantum Chromodynamics were included, the unknown aspects like hadronization being modeled phenomenologically. Each model has a set of free parameters which enables it to reproduce many distributions over a wide range of energy. In particular, the transverse momenta of the hadrons with respect to the jet direction, main subject of this paper, has been accurately parameterized. Nevertheless, there is no fundamental understanding of the matter [3].

One alternative approach to understand the transverse momentum distribution of the charged hadrons produced in e^+e^- annihilation is to look at the whole process in a macroscopic way, trying to extract information through a thermodynamical treatment, not taking into account the microscopy interactions between partons, governed by the Quantum Chromodynamics [1]. Fermi [4] was the pioneer in this kind of approach, followed by Hagedorn [5]

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[1]We used the total transverse momentum instead of p_t^{in} and p_t^{out} because we are looking at the global characteristics of the events

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who provided the most consistent and well succeeded model. Nowadays, the thermodynamical formalism is widely used in heavy ions collisions [6] and has also been applied in e^+e^-, proton-proton (pp) and proton-antiproton ($p\bar{p}$) interactions to determine the multiplicity of hadrons produced, with noteworthy results [7, 8, 9].

2   Statistical approach

According to Hagedorn, a high energy collision produces an *unlimited* and *undetermined* number of different excited hadron *fireballs* that reach a thermodynamical equilibrium. In this relativistic statistical *Ansatz*, not only the number of particles but also the number of available types of particles itself grows with energy. The immediate and important consequence of this production mechanism is that *the temperature is independent of the primary energy* [5]. A similar statement was also made, a few years later, by Field and Feynman [10]. One should expect that this temperature governs the transverse momentum (p_t) distribution of the outgoing particles with respect to the jet axis.

In a modern Ansatz [7], one can assume that the thermodynamical equilibrium is reached in each jet produced in $e^+e^- \rightarrow hadrons$ interaction. We consider in this paper that the thermalization begins just after the hadronization process takes place and finishes when the hadrons are no longer interacting. Since the range of this interaction is of order of a few Fermi, the resonances produced, with a typical lifetime of order of 10^{-23} s, should decay while the particles are still interacting. Thus, the particles produced from these decays should also participate in the thermalization process. As most charged particles observed in the detector are produced prompt or through the decay of hadronic resonances, we could expect that the charged particles observed in the detectors have their distributions dictated by the previous 'thermodynamical' equilibrium.

It is then important to determine the thermodynamical properties of the event, without any kinematical effect caused by the fast relative motion between the hadrons. Using the Boltzmann-Gibbs statistics in this large grand-canonical ensemble for a relativistic gas, the transverse momentum distribution with temperature T_0, can be written as (see Hagedorn [5]):

$$\frac{1}{\sigma}\frac{d\sigma}{dp_t} = cp_t \int_0^\infty dp_l \exp\left(-\frac{1}{T_0}\sqrt{p_l^2 + \mu^2}\right), \tag{1}$$

where p_l means the longitudinal momentum, $\mu^2 \equiv p_t^2 + m^2$ and m is the mass $(m \ll p_t)$ [5]. After the integration indicated in eq. (1) and for $p_t/T_0 \gg 1$ we obtain an exponential asymptotic distribution for the transverse momentum [5, 6]:

$$\frac{1}{\sigma}\frac{d\sigma}{dp_t} \approx cp_t^{\frac{3}{2}} \exp\left(-\frac{p_t}{T_0}\right). \tag{2}$$

As the temperature should not depend on the center-of-mass energy (E_{CM}), we can expect that eq. (2) presents the same shape for all e^+e^- energies spectrum varying only the parameter c, which depends on the average multiplicity of the events and consequently on the E_{CM}.

The measured transverse momentum distributions of charged hadrons with respect to the jet axis, defined as the sphericity axis [11, 12], for center-of-mass energies in the range of 14 GeV to 161 GeV are displayed in figure 1. One notices the clear experimental deviation, for high values of p_t, from the exponential behavior predicted by eq. (2) and indicated by a dotted line. Notice that we could also get decent fits for low values of data point of p_t $(p_t < 1.5$ GeV$)$ using the eq. (2), but in this case the best values for the parameter T_0 would clearly increase with energy, a feature which disagrees with Hagedorn's physically reasonable assumption.

3   Alternative statistical approach

The Boltzmann-Gibbs statistics, used to derive eq. (2), is suitable for an enormous number of systems, but it is known that it could lead to unphysical results for some systems that present long-range interactions (LR) [2]. In fact, LR interaction is the usual interpretation for the intermittency phenomena [13], large particle density fluctuations [14, 15], present in hadroproduction. Long-range (LR) interactions can occur in many situations, particularly in processes where the interacting volume is of order of the dimension of the

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[2]Note that we are using the expression - long range interactions - in the statistical mechanical sense, meaning interaction among particles that are far away one of the other. This situation include both cases, soft and hard interactions in QCD.

hadrons themselves, resulting in a high hadronic density. In this way the gradual deviation of the observed distribution from the exponential function, shown in figure 1, could be interpreted as a gradual increase of the multiplicity and the hadronic density in the thermalization mechanism.

Therefore it is useful, to look for another statistics that, unlike the Boltzmann-Gibbs, could treat these systems. Recently Tsallis [16] proposed a generalized statistics whose formalism [16, 17] has been able to describe many systems having as common features the presence of LR interactions and/or non-Markovian processes [18]. This formalism is based on the entropic form $S_q = (1 - Tr\rho^q)/(q-1)$ where ρ is the density matrix (state) of the system and q is a real positive parameter. When $q \to 1$ the above entropy tends to the well-known Boltzmann-Gibbs-Shannon (BGS) entropy $S = -Tr\rho \log \rho$. In this sense the S_q entropy extends the BGS entropy to the nonextensive cases corresponding to $q \neq 1$. In fact, in this case the additivity property of the entropy fails because it appears a new term proportional to $q - 1$. For example, if we look the way the composed entropy of two sub-systems A and B, S_{A+B}, is related to the entropy of each of the two sub-systems, S_A and S_B, when the sub-systems A and B are statistically independents, one gets the expression $S_{A+B} = S_A + S_B + (1-q)S_A S_B$. It is easy here to see that the case $q = 1$ reobtains the standard result (additivity) from Boltzmann-Gibbs entropy [17, 20]. The extremization of the S_q entropy with adequate constraints lead us to a probability distribution that exhibits a power-law decay (instead of the exponential decay presented by the BGS entropy). Examples of application of this statistics are available in several papers [18], whose studied systems are, in general, nonextensive. Thus, assuming a thermal equilibrium of a relativistic gas (the *fireball*) that obeys the Tsallis statistics we obtain, observing the derivation made by [19], the following equation for the transverse momentum distribution (which replaces eq. (1)):

$$\frac{1}{\sigma}\frac{d\sigma}{dp_t} = cp_t \int_0^\infty dp_l [1 - \frac{1-q}{T_0}\sqrt{p_l^2 + \mu^2}]^{\frac{q}{1-q}} , \qquad (3)$$

where q is related to the degree of non-extensivity of the system [16, 17, 20]. T_0, p_l, c and μ have the same meaning as in the previous equations. The Boltzmann-Gibbs limit - eq. (1) - is achieved as $q \to 1$. The exact expression of the transverse momentum distribution, eq. (3), is written in the appendix. With this expression we fit the experimental data leaving q, T_0 and c as free parameters and the best fits are presented in the figure 1. We can see

Experiment	Energy (GeV)	q	T_0(GeV)
TASSO	14	1.020 $\pm$0.005	0.130$\pm$0.002
TASSO	34	1.1225 $\pm$0.0006	0.1152 $\pm$0.0004
DELPHI	91	1.1938 $\pm$0.0005	0.1094$\pm$0.0004
DELPHI	161	1.215 $\pm$0.002	0.110 $\pm$0.002

Table 1:

that the asymptotic behaviour of the new distribution, for larger values of p_t, represents very-well the deviation from the exponential for *all* center-of-mass energies.

In figure 2 we present q and T_0 values as functions of the e^+e^- center-of-mass energy. Their numerical values are shown in table 1. We can see that the q parameter increases smoothly with the energy, saturating around 1.2. On the other hand, the temperature has a pretty constant behaviour, $T_0 \approx 0.11$ GeV, for the higher energy range. We note that this constant value for the temperature is not an ad hoc hypothesis; rather, it was obtained by the fit, indicating that all the energy given to the e^+e^- system is being used to create new *fireballs*, as predicted by Hagedorn [5]. We note that our T_0 value is lower than the value obtained in reference [7], and this difference is probably due to the fact that, unlike us, the author considered the thermal equilibrium just after the hadronization, i.e., before the decay of the hadronic resonances. In our process, the interaction volume continue to expand, lowering the temperature, until the particles no longer interact among themselves. Also, the statistics used in both papers are different. The smooth increase of q with the center of mass energy can be understood as the expected increase of the influence of the LR interactions among the hadronic particles produced in the event.

The deviation from an exponential behaviour observed in the transverse momentum distribution in the e^+e^- interaction, is also observed for hadrons produced in other interactions like $p\bar{p}$ or heavy ions. It indicates a universal characteristic of the dynamics of hadron production, probably due to the relevance of the LR regime, related to the high hadronic density.

4 Conclusion

The thermodynamical treatment using the Tsallis statistics discussed here presented a remarkable agreement with the experimental data in more than 4 orders of magnitude in the differential cross section - emphasized with the predicted constant behavior of the temperature T_0. Therefore, the results of this approach suggest an alternative statistical treatment for the transverse momentum distribution within the accepted theory of strong interaction.

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Appendix

After some manipulation, the solution of the integral in eq. (3) was obtained by the software Mathematica 3.0 and the transverse momentum distribution can be written as:

$$
\begin{aligned}
\frac{1}{\sigma}\frac{\partial\sigma}{\partial p_t} = & \; (T^2\sqrt{\frac{T}{T+(q-1)p_t}}(-(2^{3-\frac{q}{q-1}}\left(1+\frac{T}{(q-1)p_t}\right)^{\frac{1}{2}+\frac{q}{q-1}}\left(\frac{q-1}{T}p_t\right)^3\Gamma\left(\frac{q}{q-1}\right)* \\
& (4\Gamma\left(\frac{5}{2}-\frac{q}{q-1}\right)\Gamma\left(\frac{q}{q-1}-2\right)* \\
& {}_2F_1\left(\frac{q}{q-1}-2,\frac{q}{q-1},\frac{q}{q-1}-\frac{3}{2},\frac{(q-1)p_t+T}{2(q-1)p_t}\right)+ \\
& 2\Gamma\left(\frac{3}{2}-\frac{q}{q-1}\right)(\Gamma\left(\frac{q}{q-1}-2\right)* \\
& {}_2F_1\left(\frac{q}{q-1}-2,\frac{q}{q-1},\frac{q}{q-1}-\frac{1}{2},\frac{(q-1)p_t+T}{2(q-1)p_t}\right)+ \\
& \Gamma\left(\frac{q}{q-1}-1\right){}_2F_1\left(\frac{q}{q-1}-1,\frac{q}{q-1},\frac{q}{q-1}-\frac{1}{2},\frac{(q-1)p_t+T}{2(q-1)p_t}\right))+ \\
& \Gamma\left(\frac{1}{2}-\frac{q}{q-1}\right)\Gamma\left(\frac{q}{q-1}-1\right)* \\
& {}_2F_1\left(\frac{q}{q-1}-1,\frac{q}{q-1},\frac{q}{q-1}+\frac{1}{2},\frac{(q-1)p_t+T}{2(q-1)p_t}\right)))- \\
& \sqrt{2}\left(1+\frac{q-1}{T}p_t\right)\pi^2(8\left(\left(\frac{q-1}{T}\right)p_t\right)^2* \\
& {}_2F_1Regularized\left(-\frac{1}{2},\frac{1}{2},\frac{3}{2}-\frac{q}{q-1},\frac{(q-1)p_t+T}{2(q-1)p_t}\right)+ \\
& \left(1-\left(\frac{q-1}{T}\right)p_t\right)(-4\left(\frac{q-1}{T}\right)p_t* \\
& {}_2F_1Regularized\left(-\frac{1}{2},\frac{3}{2},\frac{5}{2}-\frac{q}{q-1},\frac{(q-1)p_t+T}{2(q-1)p_t}\right)+ \\
& 2\left(\frac{q-1}{T}\right)p_t{}_2F_1Regularized\left(\frac{1}{2},\frac{3}{2},\frac{5}{2}-\frac{q}{q-1},\frac{(q-1)p_t+T}{2(q-1)p_t}\right)- \\
& 3\left(1-\left(\frac{q-1}{T}\right)p_t\right)* \\
& {}_2F_1Regularized\left(\frac{1}{2},\frac{5}{2},\frac{7}{2}-\frac{q}{q-1},\frac{(q-1)p_t+T}{2(q-1)p_t}\right)))Sec\left(\frac{\pi q}{q-1}\right)))/ \\
& \left(16\sqrt{\left(\frac{q-1}{T}p_t\right)}\left(1+\frac{q-1}{T}p_t\right)^{\frac{q}{q-1}}\sqrt{\pi}\Gamma\left(\frac{q}{q-1}\right)(q-1)^2\right)
\end{aligned}
$$

where ${}_2F_1(\alpha,\beta;\gamma;z)$ means the generalized hypergeometric series, ${}_2F_1Regularized(\alpha,\beta;\gamma;z) \equiv {}_2F_1(\alpha,\beta;\gamma;z)/\Gamma(\gamma)$ and $\Gamma(x)$ is the Gamma function.
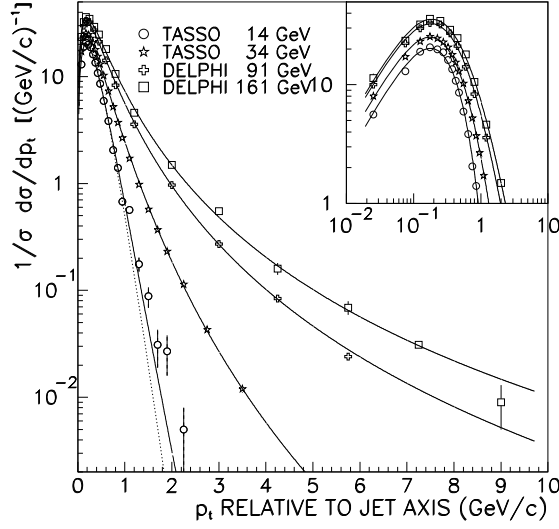
Figure 1: Transverse momentum distribution . The distribution $\frac{1}{\sigma}\frac{d\sigma}{dp_t}$ of the transverse momentum p_t of charged hadrons with respect to jet axis (defined in these experimental results as the sphericity axis) is sketched for four different experiments, whose center of mass energies vary from 14 and 34 Gev (TASSO) up to 91 and 161 Gev (DELPHI). The Hagedorn predicted exponential behaviour is shown by the dotted line. We can see that the deviation of the exponential behaviour increases when the energy increases. The continuous lines are obtained from our eq. (3) and agree very-well with the experimental data. The inset shows the transverse momentum distribution for small values of p_t.
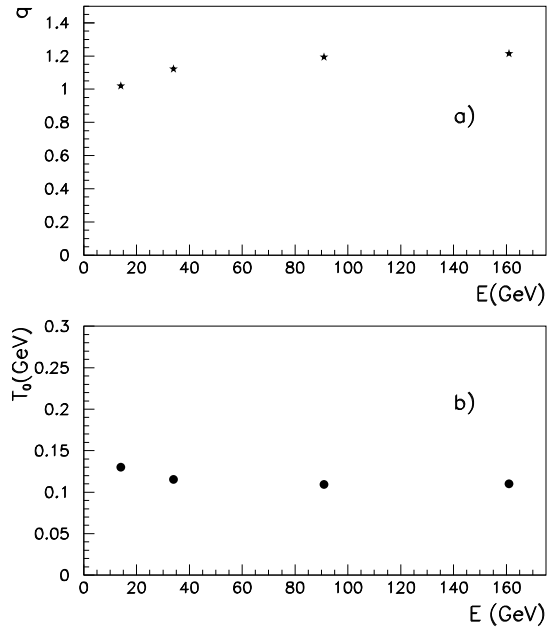
Figure 2: Variation of (a) the entropic index (q) and (b) the temperature (T_0) with the center of mass energy. The entropic index q (stars) increases slightly and the temperature (squares) remains almost constant, as predicted by Hagedorn, . The center-of-mass energies vary from 14 to 161 Gev. The error bars are smaller than the size of the symbols.